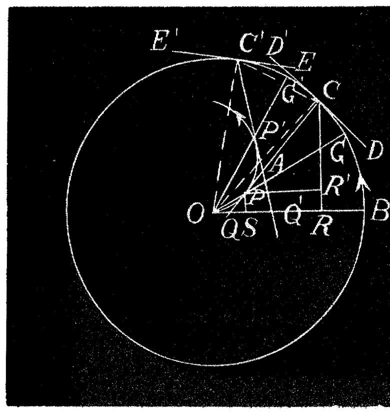
19. Proposed by A. L. FOOTE, C. E., Merrick, Queens County, New York.

A and B are in a circular room $2R=30$ feet in diameter, A being at the center and B at the circumference. B runs around at the rate of $v=600$ feet per minute and A pursues him at the rate of $u=100$ feet per minute. How long will the race last, and how far will each have traveled till B is caught?

I. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let O be the origin, OB the axis of x, B the starting point, $OB=a$, $OS=x$, $SP=y$, $OR=x_1$, $PC=m$, $OP=r$, $\angle COB=\psi$, $\angle POS=\theta$, $\angle CQS=\phi$.



v=velocity of pursued u= velocity of pursuer; s=arc OP; P, P' two consecutive points on the pursuer's path; $\frac{v}{u}=n$. Then $ns=a\psi=a\cos^{-1}\frac{x_1}{a}=$

$$a\cos^{-1}\left(\frac{x+PR'}{a}\right).$$

$PR':CR'=QS:PS$. $\therefore PR'=m\cos\phi$ $=m\frac{dx}{ds}$. $\therefore$ $ns=a\cos^{-1}\left[\frac{x+m\frac{dx}{ds}}{a}\right]$,

or $a\cos\frac{ns}{a}=x+m\frac{dx}{ds}\dots\dots(1)$. But $a^2=r^2+m^2-2mr\cos OPC=r^2+m^2$ $+2mr\cos(\phi-\theta)$. $\cos(\phi-\theta)=\frac{dr}{ds}$, $\therefore$ $a^2=r^2+m^2+2mr\frac{dr}{ds}$.

$$\therefore\quad m=\sqrt{a^2-r^2+r^2\left(\frac{dr}{ds}\right)^2}-r\frac{dr}{ds},\ \text{but}\ \frac{ds}{dr}=\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2},$$

$$\frac{dy}{dx}=\frac{\sin\theta\frac{dr}{d\theta}+r\cos\theta}{\cos\theta\frac{dr}{d\theta}-r\sin\theta}. \qquad x=r\cos\theta,\ y=r\sin\theta,\ r=\sqrt{(x^2+y^2)}.$$

$$\therefore\ \frac{d\theta}{dr}=\frac{x\frac{dy}{dx}-y}{\sqrt{(x^2+y^2)}\left(x+y\frac{dy}{dx}\right)},\quad \text{and}\ \frac{ds}{dr}=\frac{\sqrt{(x^2+y^2)\left[1+\left(\frac{dy}{dx}\right)^2\right]}}{x+y\frac{dy}{dx}}.$$

$$\therefore\ m=\frac{\sqrt{a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]-\left(y-x\frac{dy}{dx}\right)^2}-\left(x+y\frac{dy}{dx}\right)}{1+\left(\frac{dy}{dx}\right)^2}.$$

$$\therefore\ a\cos\frac{ns}{a}=x+\frac{\sqrt{a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]-\left(y-x\frac{dy}{dx}\right)^2}-\left(x+y\frac{dy}{dx}\right)}{1+\left(\frac{dy}{dx}\right)^2}.$$

If $CR=y_1$, $ns=a\sin^{-1}\frac{y_1}{a}=a\sin^{-1}\left(\frac{y+CR'}{a}\right)$,

but $CR'=m\sin\phi=m\frac{dy}{ds}$. $\therefore\ a\sin\frac{ns}{a}=y+m\frac{dy}{ds}\ldots.(2)$.

$$\therefore\ a\sin\frac{ns}{a}=y+\frac{\sqrt{a^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}-\left(y+x\frac{dy}{dx}\right)^2}-\left(x+y\frac{dy}{dx}\right)}{\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}\left\{1+\left(\frac{dx}{dy}\right)^2\right\}}}.$$

The solution of either of these differential equations (if possible) give the rectangular equation to the path of the pursuer After we know the equa- to the curve we can find its length, from which we know the time and distance. If the pursuer's velocity is less than that of the pursued the race will last an infinitely long time, or the *pursued* will catch the *pursuer* and thus end the race.

II. **Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Let O and B represent the starting-points of A and B, then will P and C represent their positions at any time after starting. Make $OB=R=15$ feet, $n=v\div u=6$, $\angle BOC=\psi$, $\angle BOG=\theta$, and $BQC=\phi$. Obviously the polar and the Cartesian co-ordinates of the point P located on the required curve, are respectively $(r,\ \theta)$ and $(x,\ y)$. Hence $dy\div dx=\tan\phi$,

$OQ=y(dx\div dy)=y\cot\phi$, $\angle OPQ=(\phi-\theta)$, and $POC=(\psi-\theta)$.

Now $PQ=\left(\frac{\sin(\psi-\theta)}{\sin(\phi-\theta)}\right)OQ=y\operatorname{cosec}\phi\ldots.(1)$. That is, $\frac{\sin(\psi-\theta)}{\sin(\phi-\theta)}$

$=\tan\phi\operatorname{cosec}\phi\ldots.(2)$.

$\therefore\ \sin\psi$

$$=\frac{-(\tan\phi-\tan\theta)\pm\sqrt{\{(\tan\phi-\tan\theta)^2+(1+\tan^2\theta)[2\tan\phi\tan\theta-\tan^2\phi]}}{1+\tan^2\theta};$$

and $\psi=\sin^{-1}\left(\frac{(\tan\phi-\tan\theta)\pm\tan\theta\sqrt{(1+2\tan\phi\tan\theta-\tan^2\phi)}}{1+\tan^2\theta}\right)$,

$$=\sin^{-1}\left(\frac{(\tan\phi-\tan\theta)\pm\tan\theta\sqrt{[(1+\tan^2\theta)-(\tan\phi-\tan\theta)^2]}}{1+\tan^2\theta}\right).$$

But $s=\int\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}dx$; $\tan\theta=y\div x$; and from the problem, $ns=R\psi\ldots.(3)$.

By obvious transformations, we have from (3) $\left(1+\frac{y^2}{x^2}\right)\sin\left(\frac{ns}{R}\right)=$

$\left\{\left(\frac{dy}{dx}-\frac{y}{x}\right)\pm\frac{y}{x}\sqrt{\left[\left(1+\frac{y^2}{x^2}\right)-\left(\frac{dy}{dx}-\frac{y}{x}\right)^2\right]}\right\}$, which is the Cartesian differen-

tial equation of the required curve; and this equation does not appear to be integrable. Many other differential equations of the required curve can be deduced; but all of these equations, as to their integrability, transcend the present limits of mathematical genius.

III. **Solution By JAMES McMAHON, M. A., Associate Editor of the "Annals of Mathematics", Department of Mathematics, Cornell University, Ithaca, New York.**

Let arc $BC=\nu$, $<DCQ=\mu$, P, P' two consecutive points on the curve, then $dm=C'P'-CP=C'A-CA-(AP+AP')=CC'\cos\mu-\frac{u}{v}\,CC'$.

Let $\frac{u}{v}=n'$. $\therefore\ dm=d\nu(\cos\mu-n')\ldots.(3)$. $d\mu=C'OC-CAC=\frac{d\nu}{a}-\frac{d\nu\sin\mu}{m}$.

$\therefore\ amd\mu=(m-a\sin\mu)d\nu\ldots.(4)$. From (3) and (4) by elimination and solution (if possible) the equation to the curve of pursuit is found.

IV. **Comment by M. C. STEVENS, M. A., Professor of Mathematics, Purdue University, Lafayette, Indiana.**

To solve this problem, if we let (x', y') be the co-ordinates of a point of the pursued at any moment, and (x, y) the coordinates of the pnrsuer at the corresponding moment, then we have, $x'^2+y'^2=225\ldots.(1)$. $y'-y=\frac{dy}{dx}(x'-x)\ldots(2)$

$$\sqrt{\left(\frac{dx'}{dx}\right)^2+\left(\frac{dy'}{dx}\right)^2}=6\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ldots.(3).$$

By elimination we can find a differential equation involving, x, y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$. (See Boole's Diff. Equations, page 251).

But this results in a very complicated equation, which has never, so far as I know, been solved. Now since the velocity of B is greater than that of A, A will never overtake B; hence the answer to the question is, the *time* is *infinite* and the *distance* is *infinite*. (See remarks on Curve of Pursuit in Runkle's Mathematical Monthly, Vol. I, p. 248).

PROBLEMS.

25. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, New Windsor College. New Windsor, Maryland.**

The leaf of the curve: "The Devil on Two Sticks", equation $y^4-x^4+100a^2x^2-96a^2y^2=0$, revolves around the axis of x. Deduce the expression for the volume generated.

26. **Proposed by Professor J. F. W. SCHEFFER, M. A., Hagerstown, Maryland.**

According to Bessel the ratio of the squares of the polar diameter of the earth to that of the equatorial diameter is .9933254. Find at what altitude the angle made by a body falling to the earth with a perpendicular to the surface is greatest. Find also this maximum angle.

Solutions to these problems should be received on or before November 1st.